This is set 1.

1. Julia and James pick a random integer between 1 and 10, inclusive. The probability they pick the same number can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m+n$.

2. There are 38 people in the California Baseball League (CBL). The CBL cannot start playing games until people are split into teams of exactly 9 people (with each person in exactly one team). Moreover, there must be an even number of teams. What is the fewest number of people who must join the CBL such that the CBL can start playing games? The CBL may not revoke membership of the 38 people already in the CBL.

3. An ant is at one corner of a unit cube. If the ant must travel on the box's surface, the shortest distance the ant must crawl to reach the opposite corner of the cube can be written in the form $\sqrt{a}$, where a is a positive integer. Compute a.

This is set 2.

4. Let $p(x) = 3x^2 + 1$. Compute the largest prime divisor of $p(100) - p(3)$.

5. Call a positive integer *prime-simple* if it can be expressed as the sum of the squares of two distinct prime numbers. How many positive integers less than or equal to 100 are prime-simple?

6. Jack writes whole numbers starting from 1 and skips all numbers that contain either a 2 or 9. What is the 100^{th} number that Jack writes down?

This is set 3.

7. A fair six-sided die is rolled five times. The probability that the five die rolls form an increasing sequence where each value is strictly larger than the one that preceded can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

8. Let $ABCD$ be a unit square and let E and F be points inside $ABCD$ such that the line containing $\overline{EF}$ is parallel to $\overline{AB}$. Point E is closer to $\overline{AD}$ than point F is to $\overline{AD}$. The line containing $\overline{EF}$ also bisects the square into two rectangles of equal area. Suppose $[AEFB] = [DEFC] = 2[AED] = 2[BFC]$. The length of segment $\overline{EF}$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

9. A sequence a_n is defined by $a_0 = 0$, and for all $n \geq 1$, $a_n = a_{n-1} + (-1)^n \cdot n^2$. Compute a_{100}.

This is set 4.

10. How many integers $100 \leq x \leq 999$ have the property that, among the six digits in $\lfloor 280 + \frac{x}{100} \rfloor$ and x, exactly two are identical?

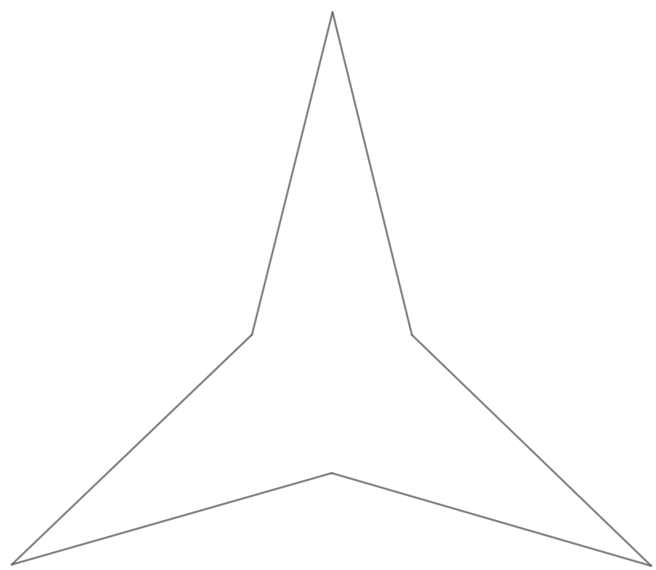
11. Compute $\sum_{x=1}^{999} \gcd(x, 10x + 9)$.

12. A hollow box (with negligible thickness) shaped like a rectangular prism has a volume of 108 cubic units. The top of the box is removed, exposing the faces on the inside of the box. What is the minimum possible value for the sum of the areas of the faces on the outside and inside of the box?

This is set 5.

13. Compute the expected sum of elements in a subset of $\{1, 2, 3, \ldots, 2020\}$ (including the empty set) chosen uniformly at random.

14. In the star shaped figure below, if all side lengths are equal to 3 and the three largest angles of the figure are 210 degrees, its area can be expressed as $\frac{a\sqrt{b}}{c}$, where a, b, and c are positive integers such that a and c are relatively prime and that b is square-free. Compute $a + b + c$.



15. Consider a random string s of 10^{2020} base-ten digits (there can be leading zeroes). We say a substring s' (which has no leading zeroes) is *self-locating* if s' appears in s at index s' where the string is indexed at 1. For example the substring 11 in the string "122352242411" is self-locating since the 11th digit is 1 and the 12th digit is 1. Let the expected number of self-locating substrings in s be G. Compute $\lfloor G \rfloor$.

This is set 6.

16. Let T be the answer to question 18. Rectangle $ZOMR$ has $ZO = 2T$ and $ZR = T$. Point B lies on segment ZO, O' lies on segment OM, and E lies on segment RM such that $BR = BE = EO'$, and $\angle BEO' = 90^\circ$. Compute $2(ZO + O'M + ER)$.

17. Let T be the answer to question 16. Compute the number of distinct real roots of the polynomial $x^4 + 6x^3 + \frac{T}{2}x^2 + 6x + 1$.

18. Let T be the answer to question 17, and let $N = \frac{24}{T}$. Leanne flips a fair coin N times. Let X be the number of times that within a series of three consecutive flips, there were exactly two heads or two tails. What is the expected value of X?

This is set 7.

19. John is flipping his favorite bottle, which currently contains 10 ounces of water. However, his bottle is broken from excessive flipping, so after he performs a flip, one ounce of water leaks out of his bottle. When his bottle contains k ounces of water, he has a $\frac{1}{k+1}$ probability of landing it on its bottom. What is the expected number of number of flips it takes for John's bottle to land on its bottom?

20. Non-degenerate quadrilateral $ABCD$ with $AB = AD$ and $BC = CD$ has integer side lengths, and $\angle ABC = \angle BCD = \angle CDA$. If $AB = 3$ and $B \neq D$, how many possible lengths are there for BC?

21. Let $\triangle ABC$ be a right triangle with legs $AB = 6$ and $AC = 8$. Let I be the incenter of $\triangle ABC$ and X be the other intersection of AI with the circumcircle of $\triangle ABC$. Find $\overline{AI} \cdot \overline{IX}$.

This is set 8.

22. Suppose that x, y, and z are positive real numbers satisfying

$$\begin{cases} x^2 + xy + y^2 = 64 \\ y^2 + yz + z^2 = 49 \\ z^2 + zx + x^2 = 57 \end{cases}$$

Then $\sqrt[3]{xyz}$ can be expressed as $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

23. Let $0 < \theta < 2\pi$ be a real number for which $\cos(\theta) + \cos(2\theta) + \cos(3\theta) + \cdots + \cos(2020\theta) = 0$ and $\theta = \frac{\pi}{n}$ for some positive integer n. Compute the sum of the possible values of $n \leq 2020$.

24. For positive integers N and m, where $m \leq N$, define

$$a_{m,N} = \frac{1}{\binom{N+1}{m}} \sum_{i=m-1}^{N-1} \frac{\binom{i}{m-1}}{N-i}.$$

Compute the smallest positive integer N such that

$$\sum_{m=1}^{N} a_{m,N} > \frac{2020N}{N+1}.$$

This is set 9.

25. Submit an integer between 1 and 50, inclusive. You will receive a score as follows:

    If some number is submitted exactly once: If E is your number, A is the closest number to E which received exactly one submission, and M is the largest unique submission, you will receive $\frac{25}{M}(A - |E - A|)$ points, rounded to the nearest integer.

    If no number was submitted exactly once: If E is your number, K is the number of people who submitted E, and M is the number of people who submitted the most commonly submitted number, then you will receive $\frac{25(M-K)}{M}$ points, rounded to the nearest integer.

26. Estimate the value of the 2020th prime number p such that $p + 2$ is also prime. If $E > 0$ is your estimate and A is the correct answer, you will receive $25 \min\left(\frac{E}{A}, \frac{A}{E}\right)^2$ points, rounded to the nearest integer. (An estimate less than or equal to 0 will receive 0 points.)

27. Estimate the number of 1s in the hexadecimal representation of 2020!. If E is your estimate and A is the correct answer, you will receive $\max(25 - 0.5|A - E|, 0)$ points, rounded to the nearest integer.