Time limit: 60 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise.
No calculators.

1. Shreyas has a rectangular piece of paper $ABCD$ such that $AB = 20$ and $AD = 21$. Given that Shreyas can make exactly one straight-line cut to split the paper into two pieces, compute the maximum total perimeter of the two pieces.

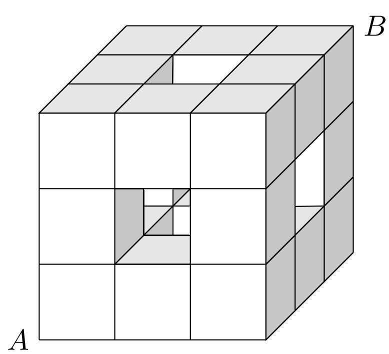
2. Compute the area of the smallest triangle which can contain six congruent, non-overlapping unit circles.

3. In quadrilateral $ABCD$, suppose that $\overline{CD}$ is perpendicular to $\overline{BC}$ and $\overline{DA}$. Point E is chosen on segment $\overline{CD}$ such that $\angle AED = \angle BEC$. If $AB = 6$, $AD = 7$, and $\angle ABC = 120^\circ$, compute $AE + EB$.

4. An equilateral polygon has unit side length and alternating interior angle measures of 15° and 300°. Compute the area of this polygon.

5. Let circles ω_1 and ω_2 intersect at P and Q. Let the line externally tangent to both circles that is closer to Q touch ω_1 at A and ω_2 at B. Let point T lie on segment $\overline{PQ}$ such that $\angle ATB = 90^\circ$. Given that $AT = 6$, $BT = 8$, and $PT = 4$, compute PQ.

6. Consider 27 unit-cubes assembled into one $3 \times 3 \times 3$ cube. Let A and B be two opposite corners of this large cube. Remove the one unit-cube not visible from the exterior, along with all six unit-cubes in the center of each face. Compute the minimum distance an ant has to walk along the surface of the modified cube to get from A to B.



7. The line l passes through vertex B and the interior of regular hexagon $ABCDEF$. If the distances from l to the vertices A and C are 7 and 4, respectively, compute the area of hexagon $ABCDEF$.

8. Let $\triangle ABC$ be a triangle with $AB = 15$, $AC = 13$, $BC = 14$, and circumcenter O. Let l be the line through A perpendicular to segment $\overline{BC}$. Let the circumcircle of $\triangle AOB$ and the circumcircle of $\triangle AOC$ intersect l at points X and Y (other than A), respectively. Compute the length of $\overline{XY}$.

9. Let $ABCD$ be a convex quadrilateral such that $\triangle ABC$ is equilateral. Let P be a point inside the quadrilateral such that $\triangle APD$ is equilateral and $\angle PCD = 30^\circ$. Given that $CP = 2$ and $CD = 3$, compute the area of the triangle formed by P, the midpoint of segment $\overline{BC}$, and the midpoint of segment $\overline{AB}$.

10. Consider $\triangle ABC$ such that $CA + AB = 3BC$. Let the incircle ω touch segments $\overline{CA}$ and $\overline{AB}$ at E and F, respectively, and define P and Q such that segments $\overline{PE}$ and $\overline{QF}$ are diameters of ω. Define the function $\mathcal{D}$ of a point K to be the sum of the distances from K to P and Q (i.e. $\mathcal{D}(K) = KP + KQ$). Let W, X, Y, and Z be points chosen on lines $\overleftrightarrow{BC}$, $\overleftrightarrow{CE}$, $\overleftrightarrow{EF}$, and $\overleftrightarrow{FB}$, respectively. Given that $BC = \sqrt{133}$ and the inradius of $\triangle ABC$ is $\sqrt{14}$, compute the minimum value of $\mathcal{D}(W) + \mathcal{D}(X) + \mathcal{D}(Y) + \mathcal{D}(Z)$.