

**Time limit:** 60 minutes.

**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

**No calculators.**

1. A cube has side length 5. Let  $S$  be its surface area and  $V$  its volume. Find  $\frac{S^3}{V^2}$ .
2. A 1 by 1 square  $ABCD$  is inscribed in the circle  $m$ . Circle  $n$  has radius 1 and is centered around  $A$ . Let  $S$  be the set of points inside of  $m$  but outside of  $n$ . What is the area of  $S$ ?
3. If  $A$  is the area of a triangle with perimeter 1, what is the largest possible value of  $A^2$ ?
4. There are six lines in the plane. No two of them are parallel and no point lies on more than three lines. What is the minimum possible number of points that lie on at least two lines?
5. A point is picked uniformly at random inside of a square. Four segments are then drawn in connecting the point to each of the vertices of the square, cutting the square into four triangles. What is the probability that at least two of the resulting triangles are obtuse?
6. A triangle  $T$  has all integer side lengths and at most one of its side lengths is greater than ten. What is the largest possible area of  $T$ ?
7. A line in the  $xy$ -plane has positive slope, passes through the point  $(x, y) = (0, 29)$ , and lies tangent to the ellipse defined by  $\frac{x^2}{100} + \frac{y^2}{400} = 1$ . What is the slope of the line?
8. What is the largest possible area of a triangle with largest side length 39 and inradius 10?
9. What is the least integer  $a$  greater than 14 so that the triangle with side lengths  $a - 1$ ,  $a$ , and  $a + 1$  has integer area?
10. A plane cuts a sphere of radius 1 into two pieces, one of which has three times the surface area of the other. What is the area of the disk that the sphere cuts out of the plane?