1. Find the value of $a$ satisfying
\[ a + b = 3 \\
3b + c = 11 \\
c + a = 61 \]

2. A point $P$ is given on the curve $x^4 + y^4 = 1$. Find the maximum distance from the point $P$ to the origin.

3. Evaluate \[ \lim_{x \to 0} \frac{\sin 2x}{e^{3x} - e^{-3x}} \]

4. Given a complex number $z$ satisfies $\text{Im}(z) = z^2 - z$, find all possible values of $|z|$.

5. Suppose that $c_n = (-1)^n(n + 1)$. While the sum $\sum_{n=0}^{\infty} c_n$ is divergent, we can still attempt to assign a value to the sum using other methods. The Abel Summation of a sequence, $a_n$, is $\text{Abel}(a_n) = \lim_{x \to 1} \sum_{n=0}^{\infty} a_n x^n$. Find $\text{Abel}(c_n)$.

6. The minimal polynomial of a complex number $r$ is the unique polynomial with rational coefficients of minimal degree with leading coefficient 1 that has $r$ as a root. If $f$ is the minimal polynomial of $\cos \frac{\pi}{7}$, what is $f(-1)$?

7. If $x,y$ are positive real numbers satisfying $x^3 - xy + 1 = y^3$, find the minimum possible value of $y$.

8. Billy is standing at $(1, 0)$ in the coordinate plane as he watches his Aunt Sydney go for her morning jog starting at the origin. If Aunt Sydney runs into the First Quadrant at a constant speed of 1 meter per second along the graph of $x = \frac{2}{5}y^2$, find the rate, in radians per second, at which Billy’s head is turning clockwise when Aunt Sydney passes through $x = 1$.

9. Evaluate the integral \[ \int_{0}^{1} \sqrt{(x - 1)^3 + 1 + x^{2/3} - (1 - x)^{3/2} - \sqrt{1 - x^2}} \, dx \]

10. Let the class of functions $f_n$ be defined such that $f_1(x) = |x^3 - x^2|$ and $f_{k+1}(x) = |f_k(x) - x^3|$ for all $k \geq 1$. Denote by $S_n$ the sum of all $y$-values of $f_n(x)$’s “sharp” points in the First Quadrant. (A “sharp” point is a point for which the derivative is not defined.) Find the ratio of odd to even terms,
\[ \lim_{k \to \infty} \frac{S_{2k+1}}{S_{2k}} \]

**P1.** Prove that for all positive integers $m$ and $n$,
\[ \frac{1}{m} \cdot \binom{2n}{0} - \frac{1}{m+1} \cdot \binom{2n}{1} + \frac{1}{m+2} \cdot \binom{2n}{2} - \cdots + \frac{1}{m+2n} \cdot \binom{2n}{2n} > 0 \]

**P2.** If $f(x) = x^n - 7x^{n-1} + 17x^{n-2} + a_{n-3}x^{n-3} + \cdots + a_0$ is a real-valued function of degree $n > 2$ with all real roots, prove that no root has value greater than 4 and at least one root has value less than 0 or greater than 2.