1. Let $ABC$ be a triangle. The angle bisectors of $\angle ABC$ and $\angle ACB$ intersect at $D$. If $\angle BAC = 80^\circ$, what are all possible values for $\angle BDC$?

2. $ABCDEF$ is a regular hexagon. Let $R$ be the overlap between $\triangle ACE$ and $\triangle BDF$. What is the area of $R$ divided by the area of $ABCDEF$?

3. Let $M$ be on segment $BC$ of $\triangle ABC$ so that $AM = 3$, $BM = 4$, and $CM = 5$. Find the largest possible area of $\triangle ABC$.

4. Let $ABCD$ be a rectangle. Circles $C_1$ and $C_2$ are externally tangent to each other. Furthermore, $C_1$ is tangent to $AB$ and $AD$, and $C_2$ is tangent to $CB$ and $CD$. If $AB = 18$ and $BC = 25$, then find the sum of the radii of the circles.

5. Let $A = (1, 0)$, $B = (0, 1)$, and $C = (0, 0)$. There are three distinct points, $P, Q, R$, such that \{A, B, C, P\}, \{A, B, C, Q\}, \{A, B, C, R\} are all parallelograms (vertices unordered). Find the area of $\triangle PQR$.

6. Let $C$ be the sphere $x^2 + y^2 + (z - 1)^2 = 1$. Point $P$ on $C$ is $(0, 0, 2)$. Let $Q = (14, 5, 0)$. If $PQ$ intersects $C$ again at $Q'$, then find the length $PQ'$.

7. Define $A = (1, 0, 0)$, $B = (0, 1, 0)$, and $P$ as the set of all points $(x, y, z)$ such that $x + y + z = 0$. Let $P$ be the point on $P$ such that $d = AP + PB$ is minimized. Find $d^2$.

8. Suppose that $A = \left(\frac{1}{2}, \sqrt{3}\right)$. Suppose that $B, C, D$ are chosen on the ellipse $x^2 + (y/2)^2 = 1$ such that the area of $ABCD$ is maximized. Assume that $A, B, C, D$ lie on the ellipse going counterclockwise. What are all possible values of $B$?

9. Let $ABC$ be a triangle. Suppose that a circle with diameter $BC$ intersects segments $CA, AB$ at $E, F$, respectively. Let $D$ be the midpoint of $BC$. Suppose that $AD$ intersects $EF$ at $X$. If $AB = \sqrt{5}$, $AC = \sqrt{10}$, and $BC = \sqrt{11}$, what is $\frac{EX}{XF}$?

10. Let $ABC$ be a triangle with points $E, F$ on $CA, AB$, respectively. Circle $C_1$ passes through $E, F$ and is tangent to segment $BC$ at $D$. Suppose that $AE = AF = EF = 3$, $BF = 1$, and $CE = 2$. What is $\frac{ED^2}{FD^2}$?

**P1.** Suppose that circles $C_1$ and $C_2$ intersect at $X$ and $Y$. Let $A, B$ be on $C_1, C_2$, respectively, such that $A, X, B$ lie on a line in that order. Let $A, C$ be on $C_1, C_2$, respectively, such that $A, Y, C$ lie on a line in that order. Let $A', B', C'$ be another similarly defined triangle with $A \neq A'$. Prove that $BB' = CC'$. (You must include a diagram with your solution).

**P2.** Suppose that fixed circle $C_1$ with radius $a > 0$ is tangent to the fixed line $l$ at $A$. Variable circle $C_2$, with center $X$, is externally tangent to $C_1$ at $B \neq A$ and $l$ at $C$. Prove that the set of all $X$ is a parabola minus a point.