1. Let \( x, y, z, w \) be integers such that \( 2^x + 2^y + 2^z + 2^w = 24.375 \). Find the value of \( xyzw \).

2. Let \( g(x) = 1 + 2x + 3x^2 + 4x^3 + \ldots \). Find the coefficient of \( x^{2015} \) of \( f(x) = \frac{g(x)}{1-x} \).

3. Find all integer solutions to
   \[
   x^2 + 2y^2 + 3z^2 = 36,
   3x^2 + 2y^2 + z^2 = 84,
   xy + xz + yz = -7.
   \]

4. Let \( \{a_n\} \) be a sequence of real numbers with \( a_1 = -1, a_2 = 2 \) and for all \( n \geq 3 \),
   \[
a_{n+1} - a_n - a_{n+2} = 0.
   \]
   Find \( a_1 + a_2 + a_3 + \ldots + a_{2015} \).

5. Let \( x \) and \( y \) be real numbers satisfying the equation \( x^2 - 4x + y^2 + 3 = 0 \). If the maximum and minimum values of \( x^2 + y^2 \) are \( M \) and \( m \) respectively, compute the numerical value of \( M - m \).

6. The roots of the equation \( x^5 - 180x^4 + 4x^3 + Bx^2 + Cx + D = 0 \) are in geometric progression. The sum of their reciprocals is 20. Compute \( |D| \).

7. Evaluate \( \sum_{k=0}^{37} (-1)^k \binom{75}{2k} \)

8. Let \( \omega \) be a primitive 7th root of unity. Find
   \[
   \prod_{k=0}^{6} (1 + \omega^k - \omega^{2k}).
   \]
   (A complex number is a primitive root of unity if and only if it can be written in the form \( e^{2k\pi i/n} \), where \( k \) is relatively prime to \( n \).)

9. Find
   \[
   \lim_{n \to \infty} \frac{1}{n^3} \left( \sqrt{n^2 - 1} + \sqrt{n^2 - 2^2} + \ldots + \sqrt{n^2 - (n-1)^2} \right).
   \]

10. Evaluate
    \[
    \int_{0}^{\pi/2} \ln (4 \sin x) \, dx.
    \]

**P1.** Suppose \( z_0, z_1, \ldots, z_{n-1} \) are complex numbers such that \( z_k = e^{2k\pi i/n} \) for \( k = 0, 1, 2, \ldots, n-1 \).

Prove that for any complex number \( z \), \( \sum_{k=0}^{n-1} |z - z_k| \geq n \).

**P2.** Let \( f(x) \) be a nonconstant monic polynomial of degree \( n \) with rational coefficients that is irreducible, meaning it cannot be factored into two nonconstant rational polynomials. Find and prove a formula for the number of monic complex polynomials that divide \( f \).